

# Beyond Hard Negative Mining:

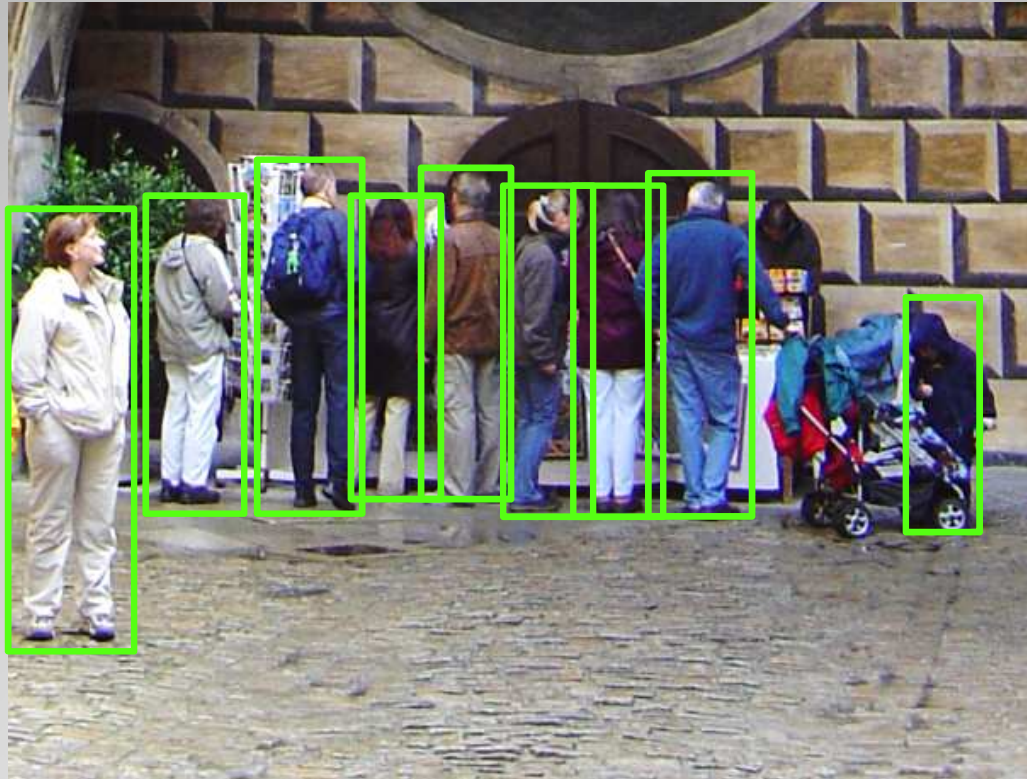
## Efficient Detector Learning via Block-Circulant Decomposition

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University of Coimbra



# Motivation



- Setting: **object detection**
- Scan image with **learned template** of dense features (e.g., HOG, SIFT, CNN...)
- Core component of many approaches

# Motivation

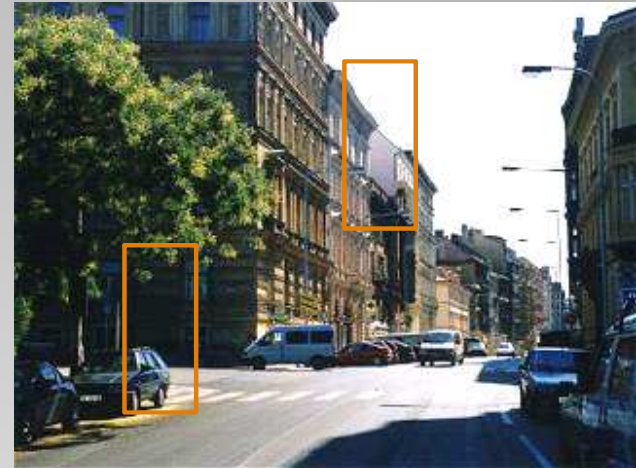
High performance usually requires  
**Hard Negative Mining.**



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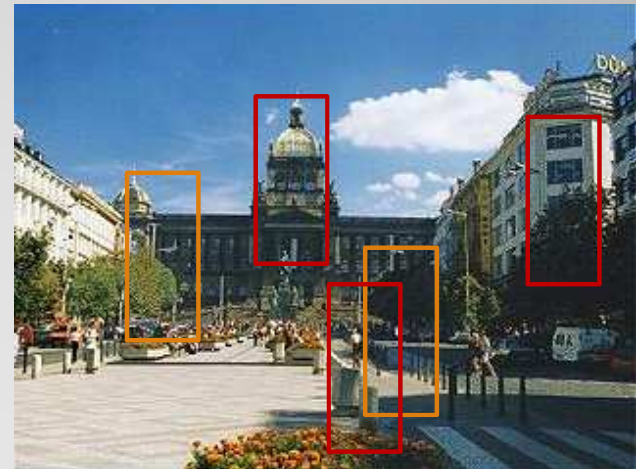
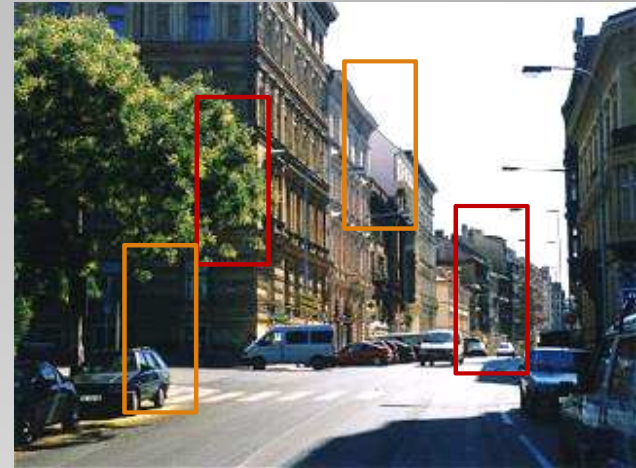
1. Train initial model (e.g., SVM) with
  1. All positive samples (not shown)
  2. Random negative samples



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1. Train initial model (e.g., SVM) with
  1. All positive samples (not shown)
  2. Random negative samples
2. Scan negative images for **false-positives**
3. Re-train using **false-positives** as additional samples



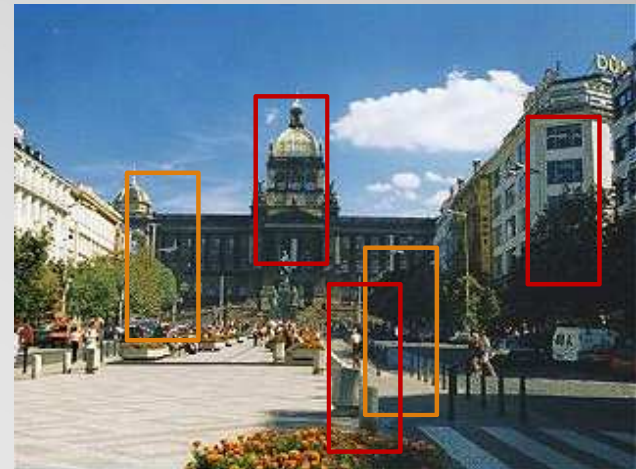
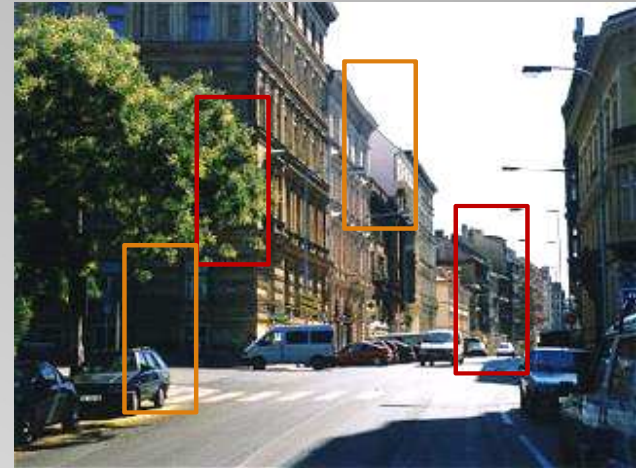
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High performance usually requires **Hard Negative Mining**.

1. Train initial model (e.g., SVM) with
  1. All positive samples (not shown)
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2. Scan negative images for **false-positives**
  3. Re-train using **false-positives** as additional samples
- (Repeat)

Several rounds are needed.  
Each round is **very expensive**.



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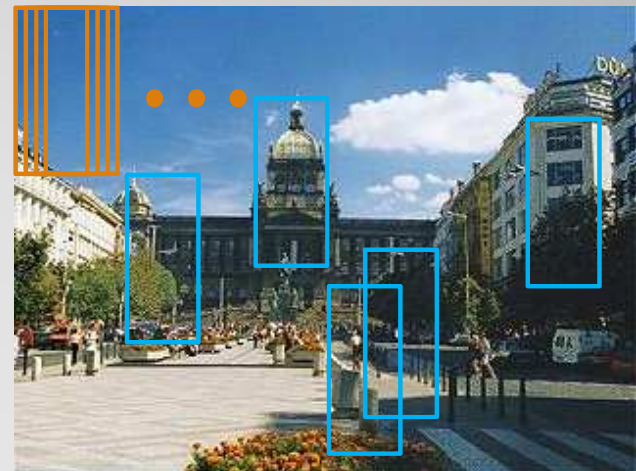
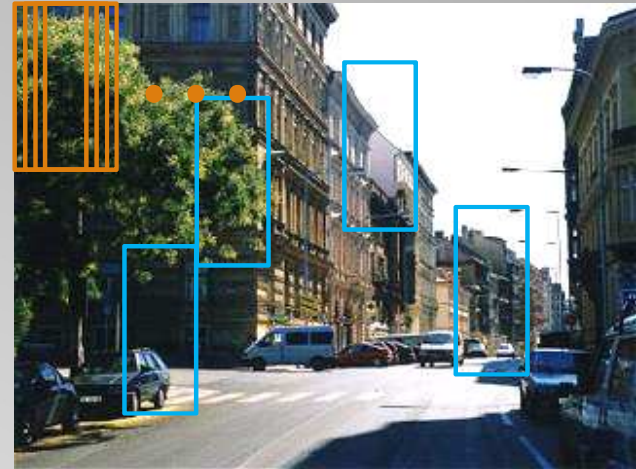
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- Consider the **full set** of all potential samples.
- Hard Negative Mining avoids working on the full set by growing an **active set** of mined samples.



# Motivation

Observation:

- Negative sets are **highly redundant**
- Pixels of **overlapping** windows are constrained to be the same

Questions:

- How does this influence a learning problem?
- Can we get rid of redundancies?

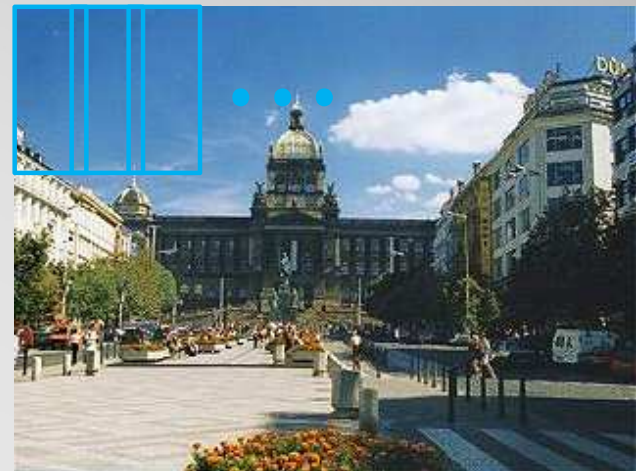


# “Bold idea”

Let's try to train with the **full negative set**.

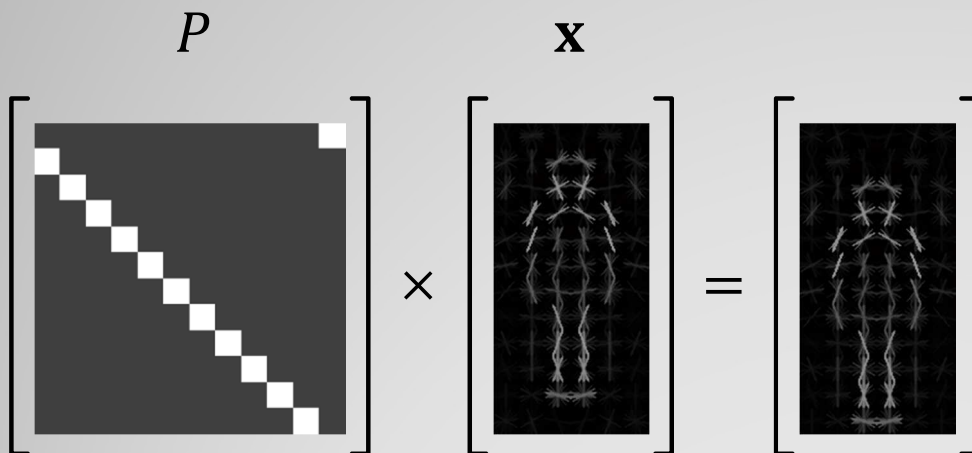
Method:

- Collect base samples in a **coarse** grid.
- Train with the finer translations **implicitly** by using a **Circulant Decomposition**.



# Cyclic shifts

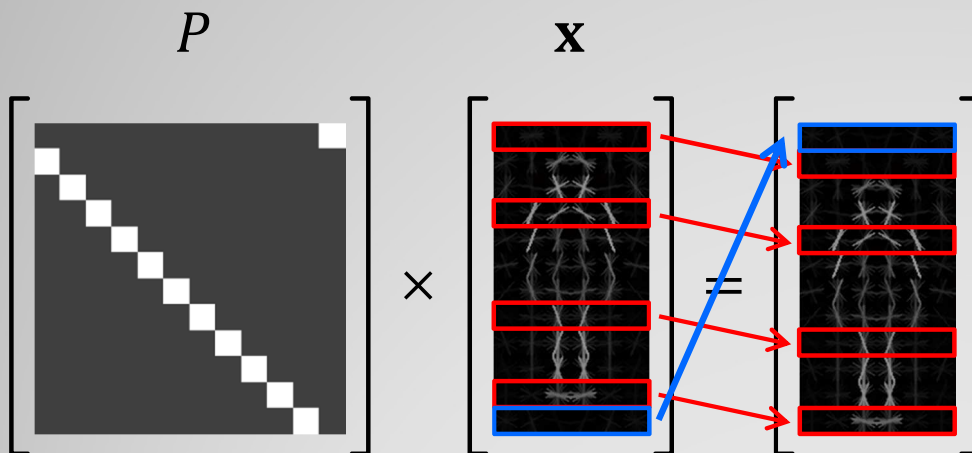
- We need a **model of image translations**.
- Idea: Apply permutation matrix  $P$  to base sample  $\mathbf{x}$ :

$$P \times \mathbf{x} = \text{shifted image}$$


$P$  represents a **cyclic shift**.

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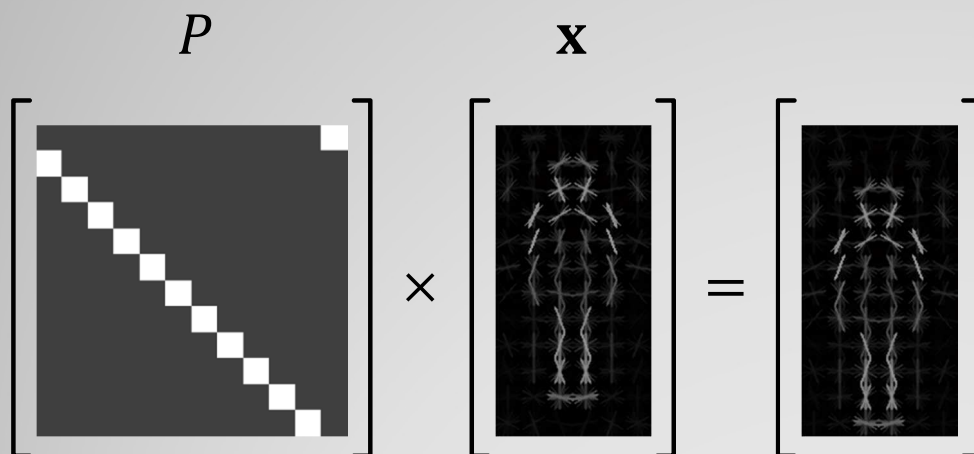


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# Cyclic shifts

- We need a **model of image translations**.
- Idea: Apply permutation matrix  $P$  to base sample  $\mathbf{x}$ :

$$P \times \mathbf{x} = \text{result}$$


$P$  represents a **cyclic shift**.

- Powers of  $P$  shift by different amounts:

$$P^u \mathbf{x}, u \in \left\{ -\frac{\text{height}}{2}, \dots, +\frac{\text{height}}{2} \right\}$$

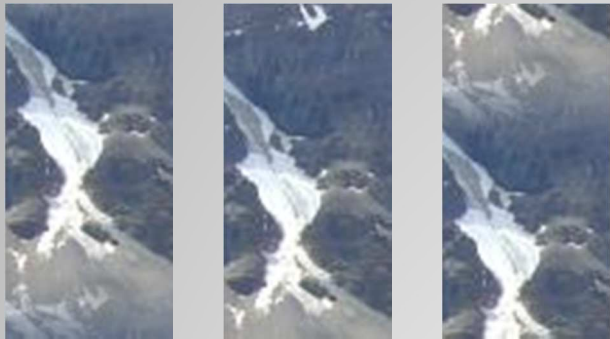
- Represents a collection of fine translations of  $\mathbf{x}$ .

(Easy to generalize to horizontal + vertical)

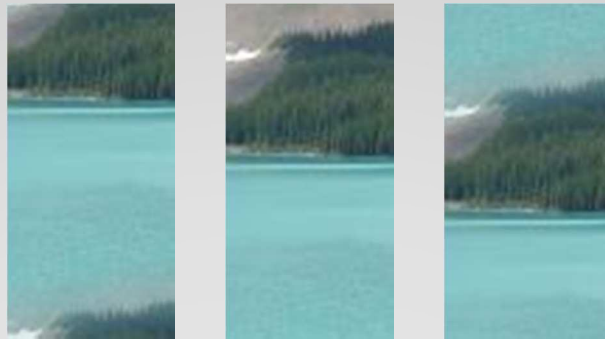
# Cyclic shifts

- Goal: implicitly train with **all shifts** of **all base samples**.

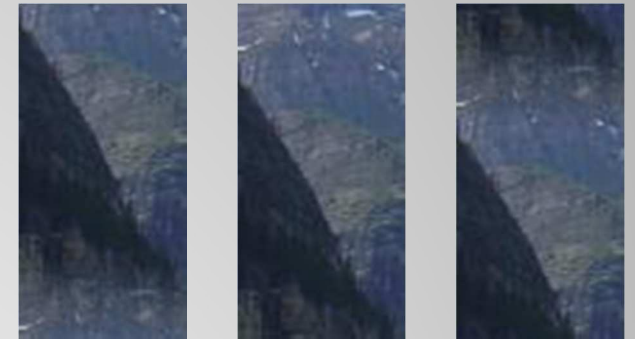
shifts of base sample  $\mathbf{x}_1$



shifts of base sample  $\mathbf{x}_2$

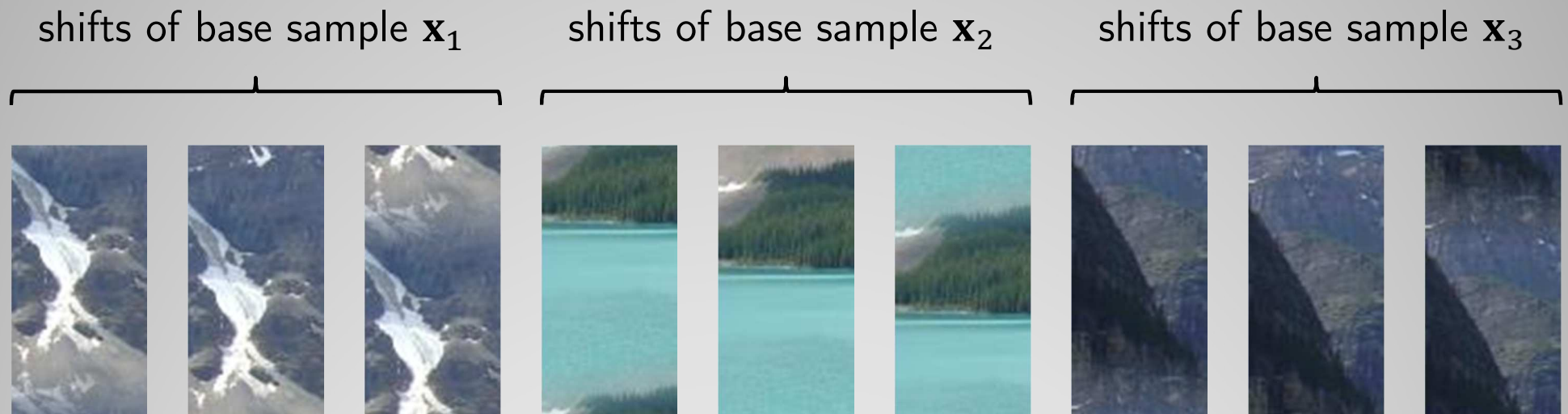


shifts of base sample  $\mathbf{x}_3$



# Cyclic shifts

- Goal: implicitly train with **all shifts** of **all base samples**.



- To see how shifted samples interact, analyze the **Gram matrix**:

$$G_{ij} = \langle \mathbf{x}_i, \mathbf{x}_j \rangle$$

(Dot-products between **pairs** of samples)

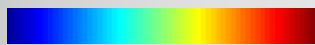
# Gram matrix

Dataset



Shift	Base sample	0		
		1	2	3
0	1	Dark Red	Orange	Blue
	2	Orange	Yellow	Dark Blue
	3	Blue	Dark Blue	Orange

$G$



dot-product

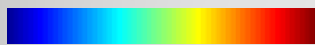
# Gram matrix

Dataset



Shift	Base sample	0			1		
		1	2	3	1	2	3
0	1	Dark Red	Orange	Blue	Cyan	Light Blue	Light Green
	2	Orange	Yellow	Dark Blue	Dark Blue	Dark Blue	Light Green
	3	Blue	Dark Blue	Orange	Orange	Light Green	Cyan
1	1	Cyan	Dark Blue	Orange	Dark Red	Orange	Blue
	2	Light Blue	Dark Blue	Light Green	Orange	Yellow	Dark Blue
	3	Light Green	Cyan	Blue	Dark Blue	Orange	Dark Blue

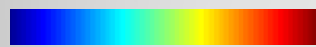
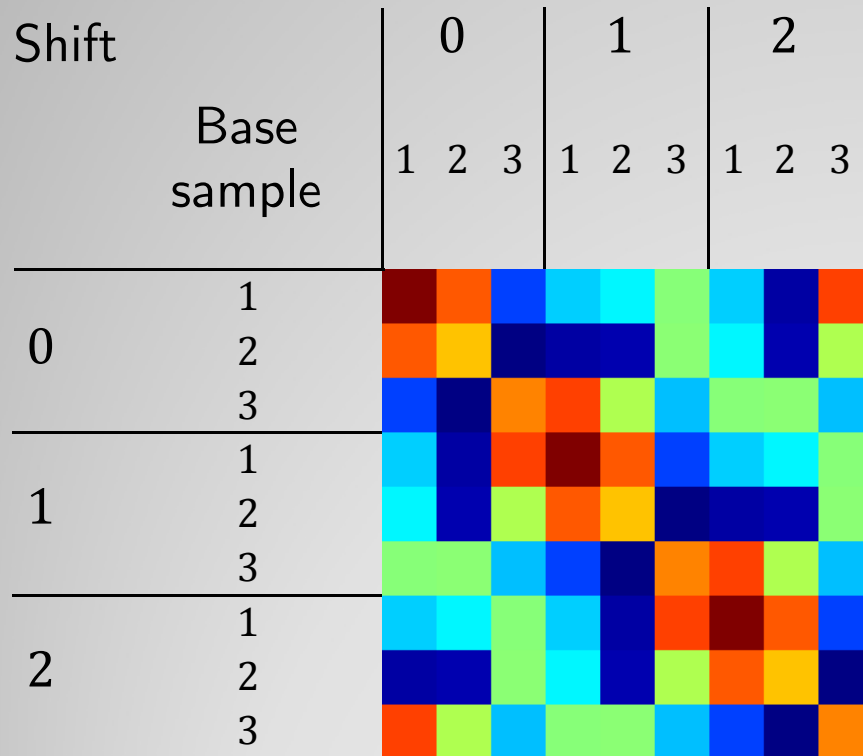
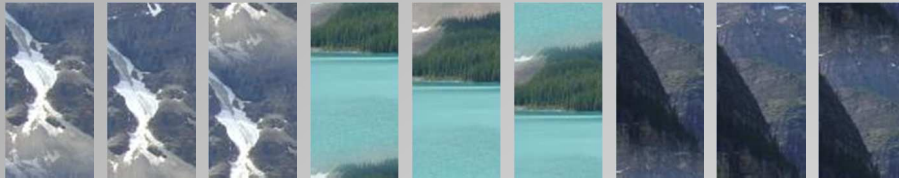
$G$



dot-product

# Gram matrix

Dataset



dot-product

$G$

# Gram matrix

Dataset



Shift	Base sample	0			1			2		
		1	2	3	1	2	3	1	2	3
0	1	█	█	█	█	█	█	█	█	█
	2	█	A	█	█	B	█	█	C	█
	3	█	█	█	█	█	█	█	█	█
1	1	█	█	█	█	█	█	█	█	█
	2	█	C	█	█	A	█	█	B	█
	3	█	█	█	█	█	█	█	█	█
2	1	█	█	█	█	█	█	█	█	█
	2	█	B	█	█	C	█	█	A	█
	3	█	█	█	█	█	█	█	█	█

$G$

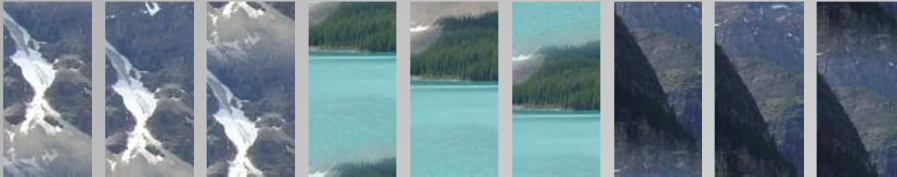
- Property #1:

$G$  is **block-circulant**

⇒ Only 1 row of blocks is unique.

# Gram matrix

Dataset



Shift	Base sample	0			1			2		
		1	2	3	1	2	3	1	2	3
0	1	■	■	■	■	■	■	■	■	■
	2	■	A	■	■	B	■	■	■	C
	3	■	■	■	■	■	■	■	■	■
1	1	■	■	■	■	■	■	■	■	■
	2	■	■	C	■	A	■	■	■	B
	3	■	■	■	■	■	■	■	■	■
2	1	■	■	■	■	■	■	■	■	■
	2	■	■	■	B	■	■	■	C	■
	3	■	■	■	■	■	■	■	■	A

$G$

- Property #1:

$G$  is **block-circulant**

⇒ Only 1 row of blocks is unique.

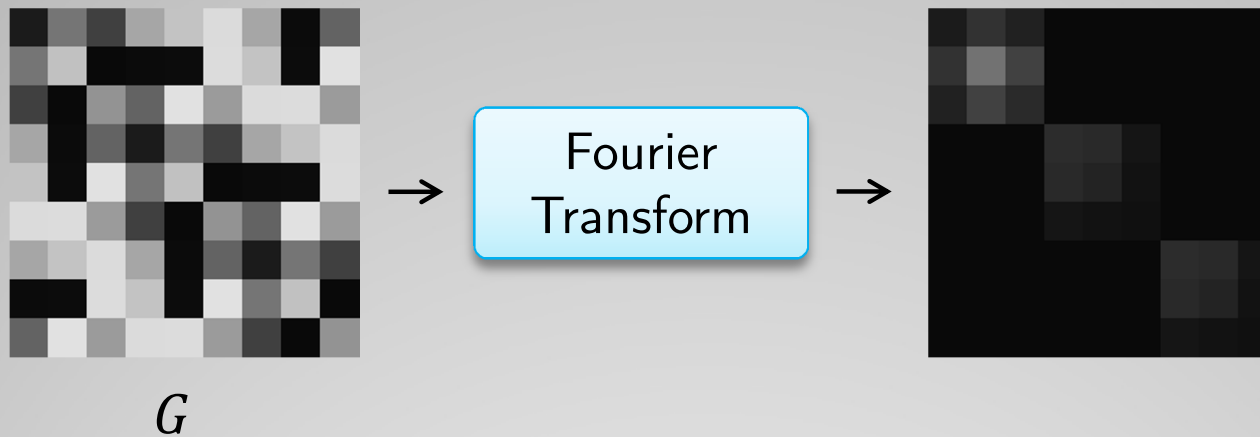
- Property #2:

Unique blocks contain the **cross-correlation** between all pairs of samples.

⇒ Becomes simple product in the Fourier domain.



# Block-diagonalization

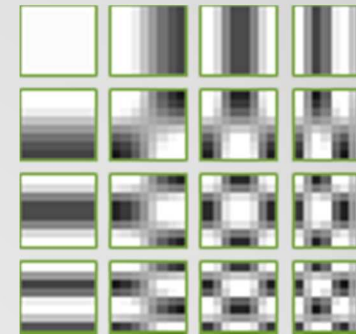


Proposed approach:

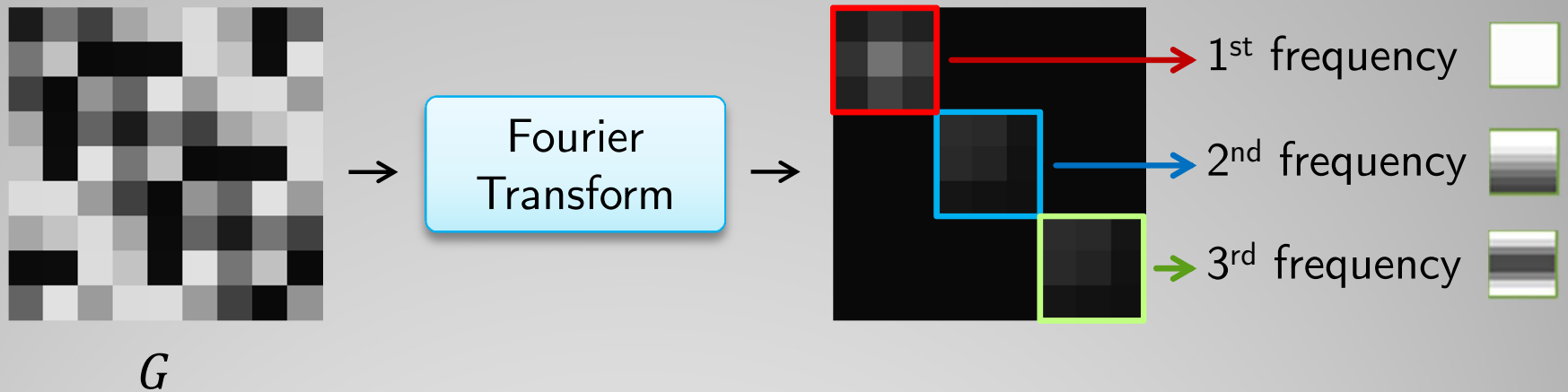
- **Fourier Transform** the samples (+ a small permutation)



Projection on **Fourier Basis** with different frequencies.



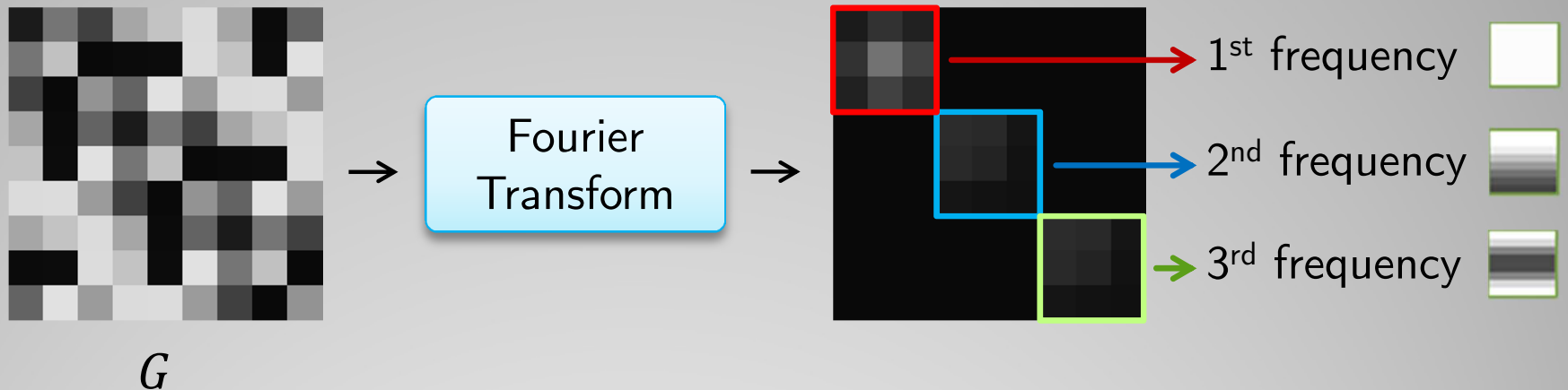
# Block-diagonalization



- **Each block** of  $G$  contains the projection on a different basis, or **Fourier frequency**.

(# of frequencies = # of spatial cells of the samples)

# Block-diagonalization

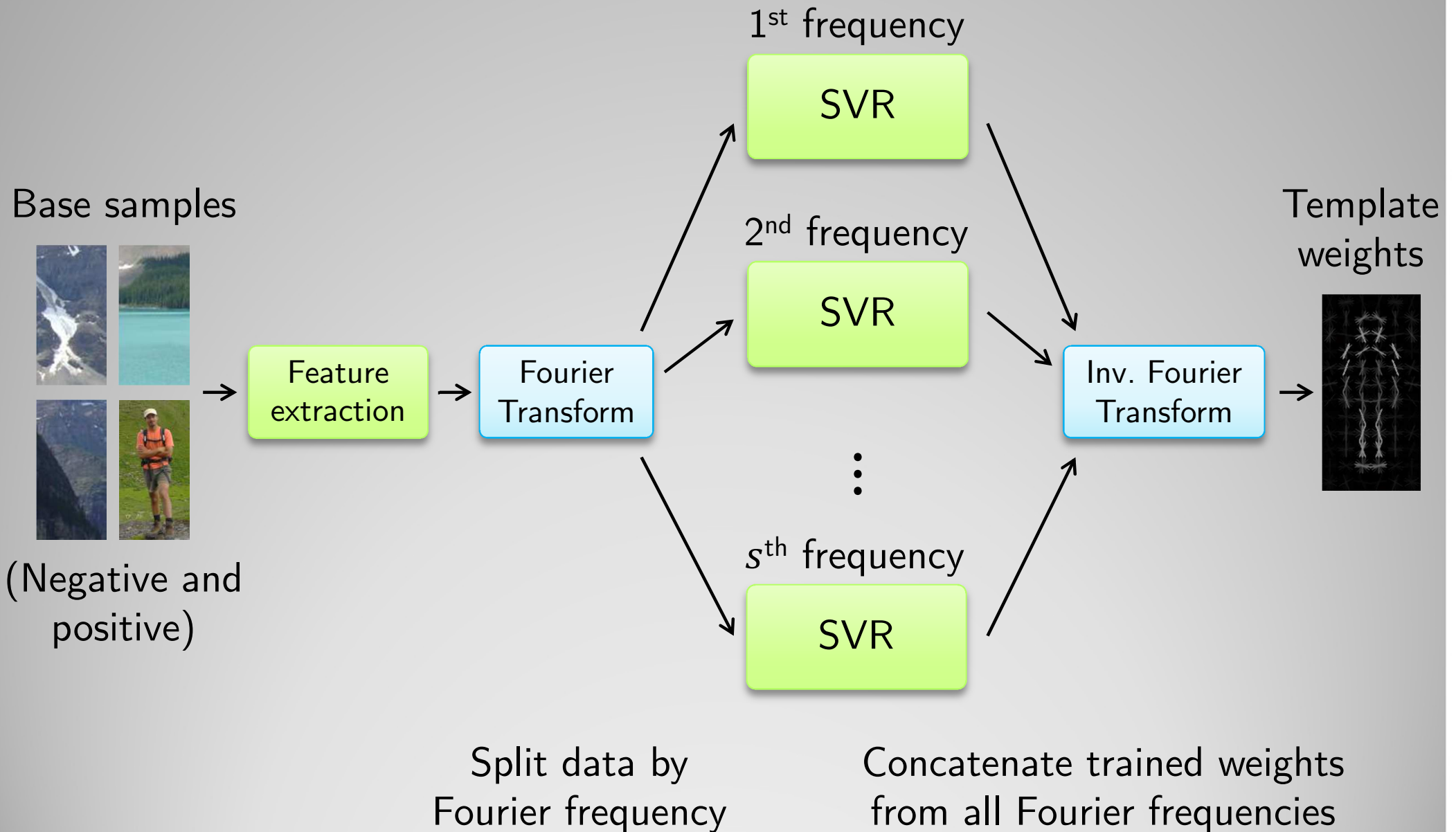


We prove all off-diagonal blocks are **zero**. → 99.5% for 18x10 HOG template

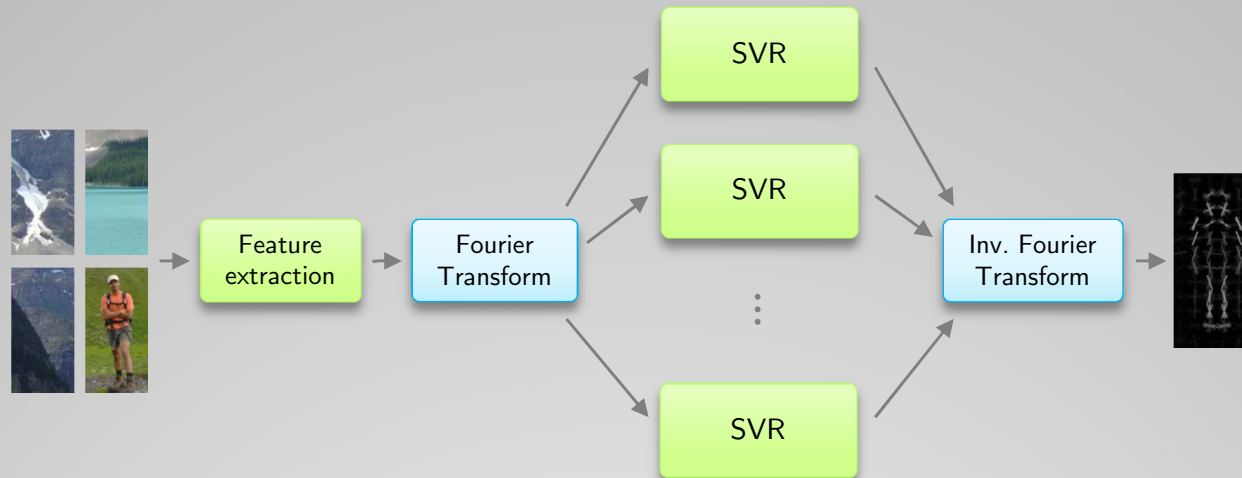


Frequencies correspond to **independent** learning problems.

# Circulant Decomposition



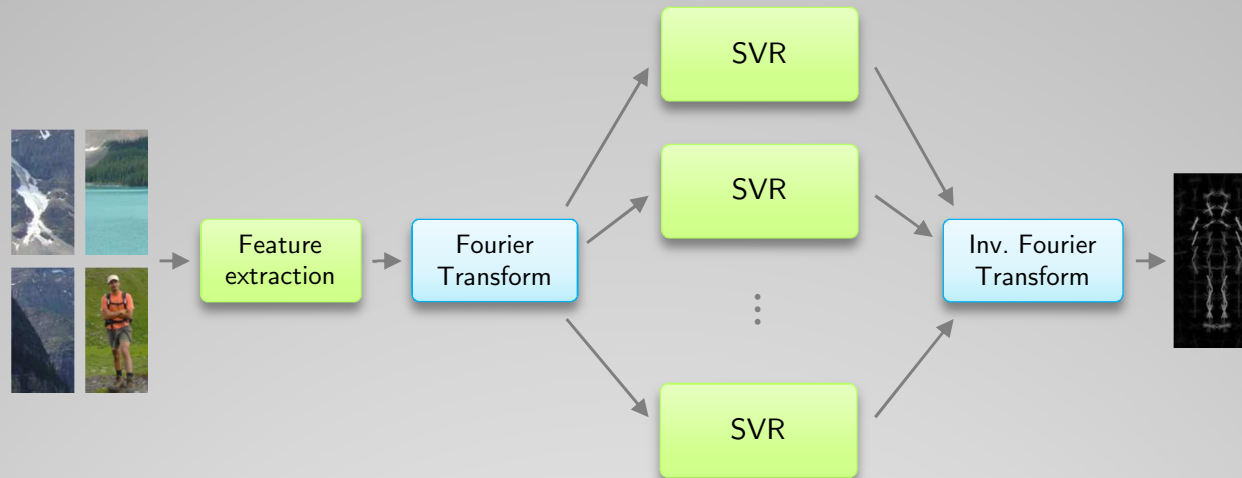
# Circulant Decomposition



Circulant  
Decomposition

- Equivalent to training with **all shifts** of the base samples.
- Surprisingly, *easier* than without shifts:
  - No shifts: one large SVR.
  - With shifts: many small SVR's.

# Circulant Decomposition

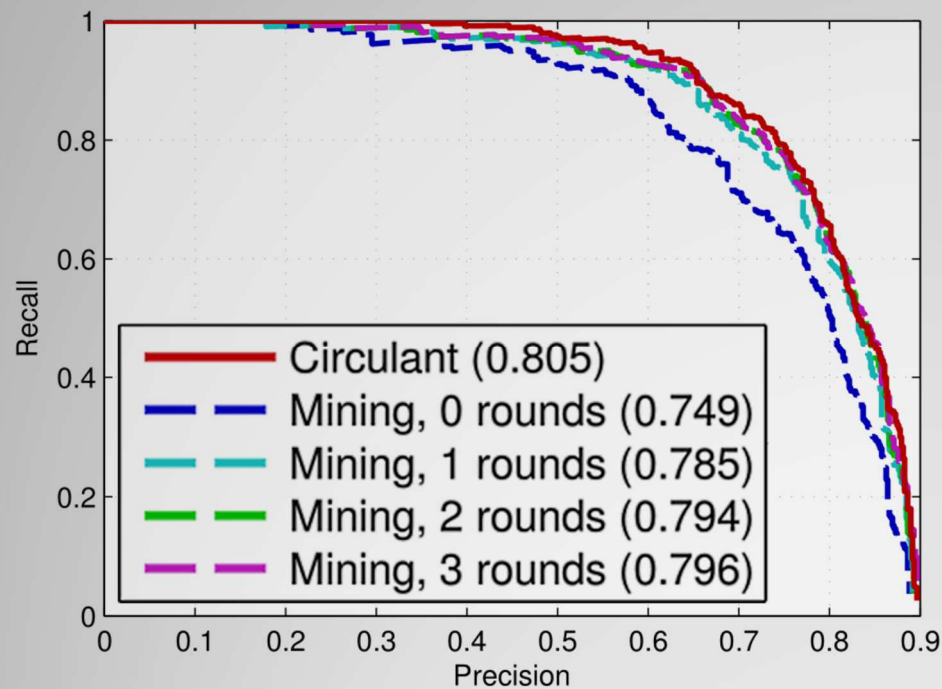


Circulant  
Decomposition

- **Closed-form**
- Sub-problems:
  - Can be solved in **parallel**
  - Use off-the-shelf SVR solvers
- **12 lines** of MATLAB code

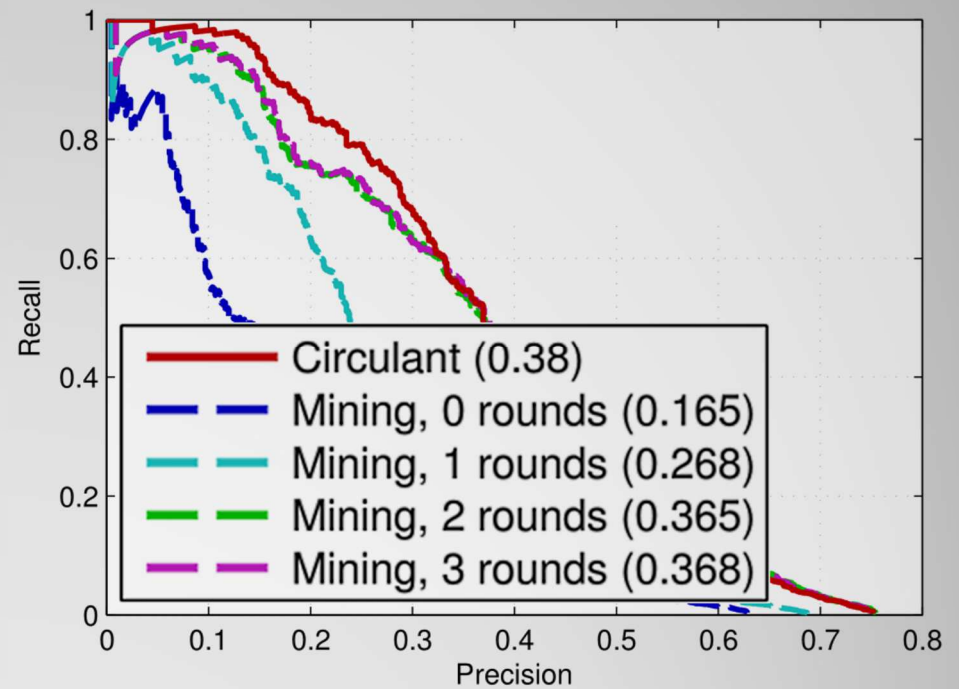
# Experiments

Single-template HOG object detection:



INRIA Pedestrians

- 1218 negative images
- $\sim 10^8$  potential samples

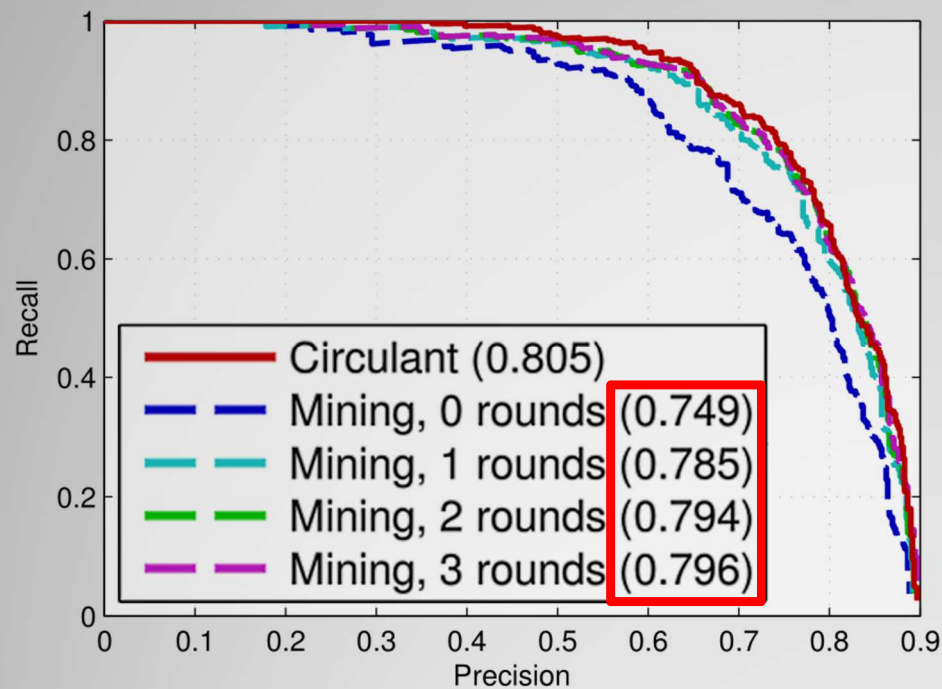


Caltech Pedestrians

- 4250 negative images
- $\sim 10^8$  potential samples

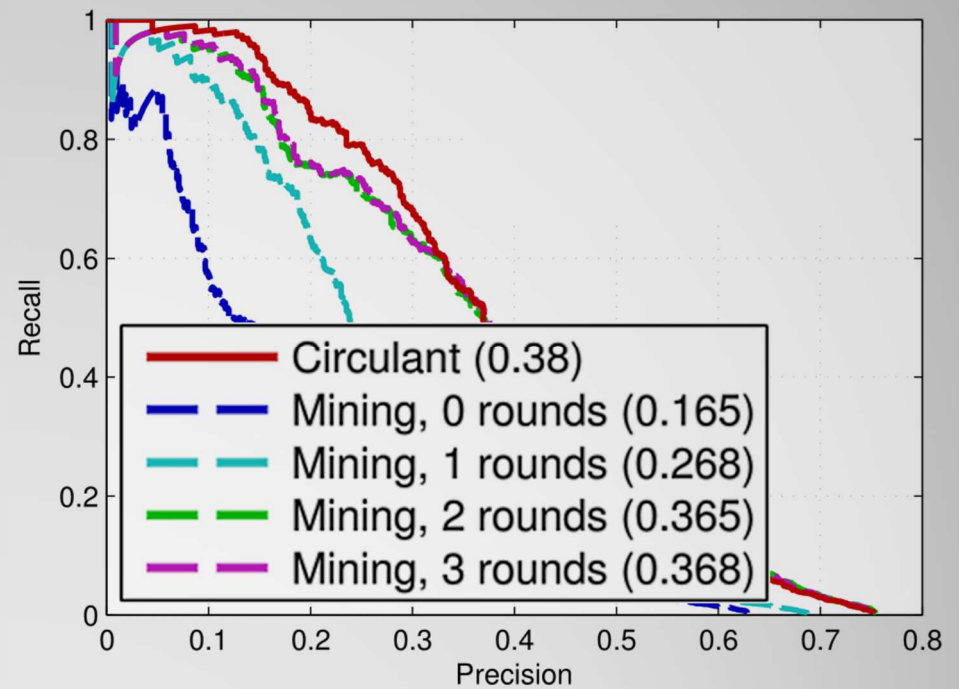
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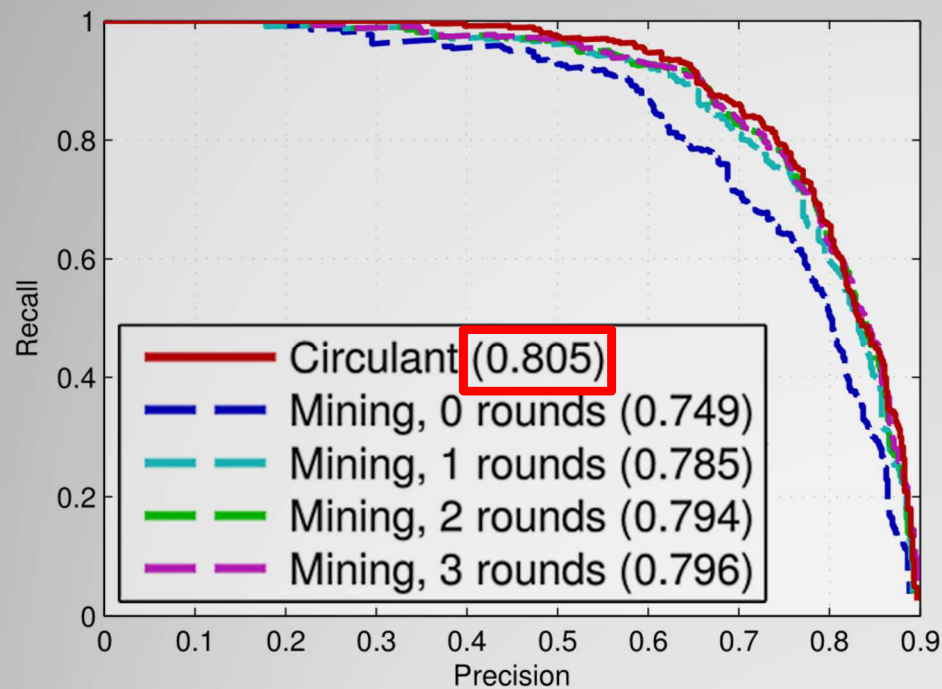
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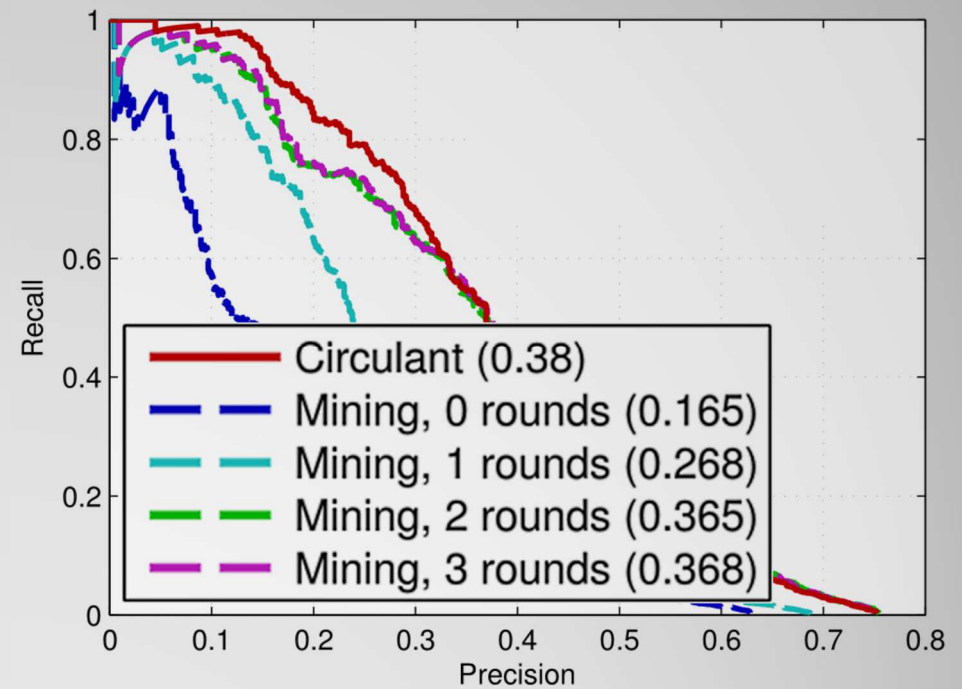
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Single-template HOG object detection:

Rounds		Mining				Circulant
		0	1	2	3	<b>0</b>
Time (s)	INRIA	7	159	312	463	<b>35</b>
AP		0.749	0.785	0.794	0.796	<b>0.805</b>
Time (s)	Caltech	12	646	1272	1901	<b>139</b>
AP		0.165	0.268	0.365	0.368	<b>0.380</b>

**~14x speed-up**

# Experiments

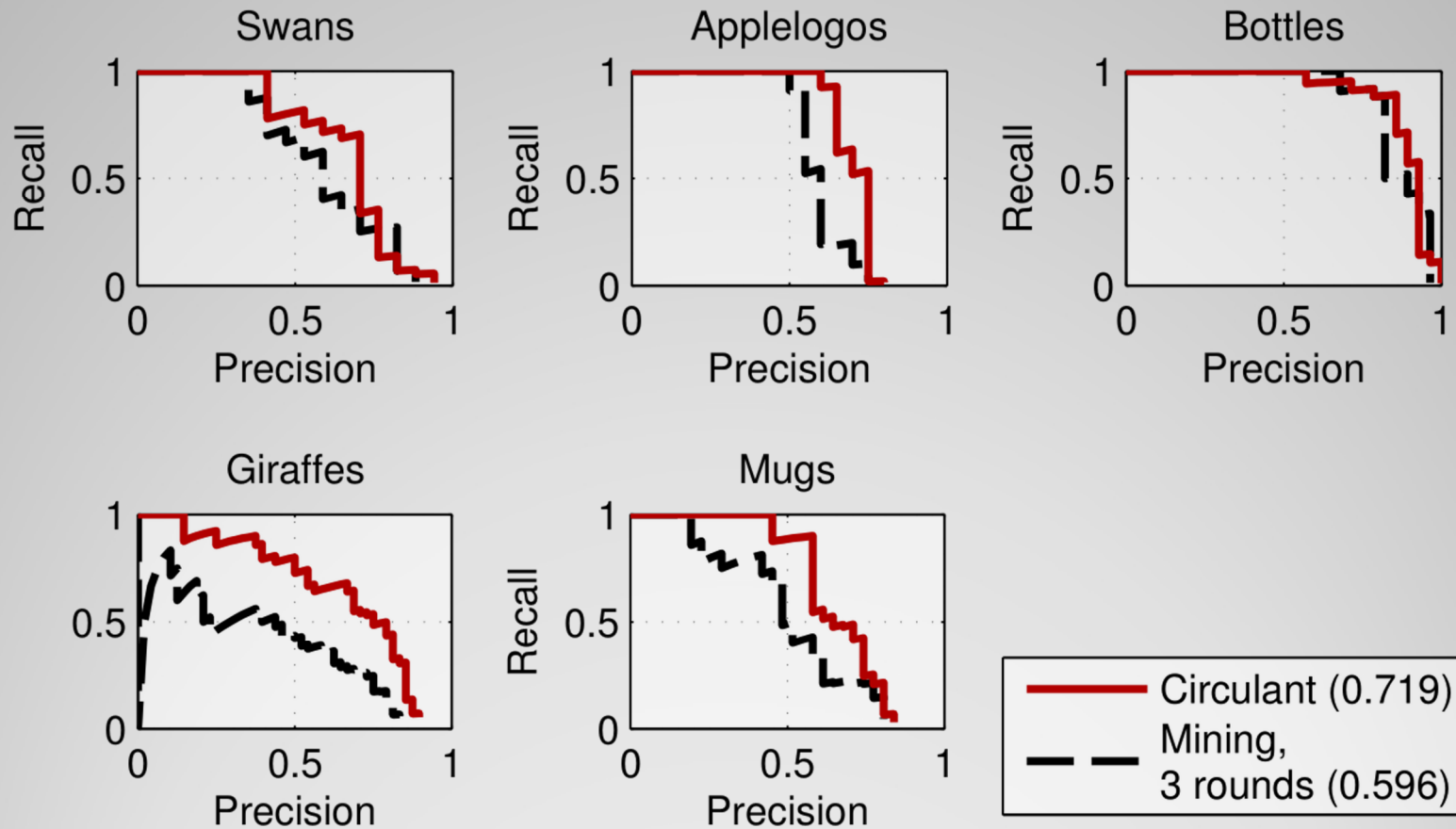
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# Experiments

Single-template HOG object detection:



ETHZ Shapes

# Conclusions

- Hard negative mining can be replaced with **non-iterative** training.
- There is a **rich intrinsic structure** in the problem.

- Circulant Decomposition:
  - Closed-form
  - Parallel, small sub-problems
  - Off-the-shelf SVR solvers
  - **12 lines** of MATLAB code

⇒ {

- ~**14x speed-up**
- Same/better performance

- Theoretic development:
  - Link between general **learning algorithms** and specialized Fourier **signal processing**.